Incorporating Longevity Risk and Medical Information into Life Settlement Pricing

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ABSTRACT

A life settlement is a financial transaction in which the owner of a life insurance policy sells her policy to a third party. We present an overview of the life settlement market, exhibit its susceptibility to longevity risk, and discuss it as part of a new asset class of longevity related securities. We discuss pricing where the investor has information concerning the expected life expectancy of the insured as well as perhaps other medical information obtained from a medical underwriter. We show how to incorporate this information into the investor's valuation in a rigorous and statistically justified manner. To incorporate medical information, we apply statistical information theory to adjust a pre-specified standard mortality table so as to obtain a new mortality table that exactly reflects the known medical information. We illustrate using several mortality tables including a new extension of the Lee- Carter model that allows for jumps in mortality and longevity over time. The information theoretically adjusted mortality table has a distribution consistent with the underwriter's projected life expectancy or other medical underwriter information and is as indistinguishable as possible from the pre-specified mortality model. An analysis using several different potential standard tables and medical information sets illustrates the robustness and versatility of the method.

Key words: Life Settlement, Asset Class, Double Exponential Jump Diffusion Model, Information Theoretic Dynamic Pricing.

1. INTRODUCTION

While the effect of longevity risk is traditionally thought of in terms of its impact on pensions, social security systems and the solvency of corporate defined benefit plans, there is another market that is vulnerable to longevity risk, perhaps even more than the above areas, namely the life settlement (and life securitization) market. A life settlement is a financial arrangement whereby the third party (or investor) purchases a life insurance policy from the person who originally purchased the life insurance policy. This third party pays the insured an amount greater than the cash surrender value of the policy -- in effect, the trade-in value of the policy as determined by the originating insurance company[2] -- but less than the face value (or the death benefit). The investor also agrees to pay future premium payments in exchange for the right to collect the death benefit upon the death of the insured.

A life settlement can be a win-win situation, as the investor can obtain a return on their initial investment and premium payments once the death benefit becomes payable (assuming the insured does not live too much longer than expected when setting the purchase price) and the owner of the policy obtains more money than they would if they had surrendered the policy for its cash value or allowed it to lapse (*cf.*, Doherty and Singer 2003)[3]. Life settlements are a part of the newly emerging and growing asset class of longevity and mortality related financial instruments providing investors with assets essentially uncorrelated with other market related assets in the investors' portfolio, hence increasing diversification effects (*cf.*, Cowley and Cummins. 2005).

This life settlement market has a vulnerability to longevity risk as increased longevity implies longer periods during which investors are paying premiums prior to collecting their money, and hence there is a potential for losing money, going bankrupt, or seeing a severe reduction in the expected return on the investment. The

[2] The cash value of the policy is also known as the non-forfeiture value since this is the least amount the insurer can pay to a surrendering policy holder. Formula for calculating the cash value can be found in Bowers et al (1997).

[3] According to a 2001 study of lapse rates for life insurance policies by the Life Insurance Marketing Research Association (Purushotham 2001), the majority of life insurance policies that lapse (4.3% of policies) are surrendered for cash value. By four years after purchase, more than 75% of policies that lapse are full surrenders of the policy, leaving potentially millions of dollars on the table to be reclaimed by life insurers in the absence of a secondary market for life insurance policies vending at more than the face value, a secondary market provided by the life settlement market.

rise and fall of the viatical settlement market, from whence the life settlement market arose, illustrates these dangers and susceptibility to increases in longevity. A brief history of the viatical settlement market illustrates the longevity risk inherent in the life settlement market.

The practice of buying and selling "viatical settlements" began in the late 1980s when a devastating medical AIDS epidemic presented a financial shock to thousands of previously healthy Americans (*e.g.*, see Stone and Zissu 2006). Due to the extremely high medical costs associated with treatments for this disease and due to the difficulty for HIV positive individuals to work and maintain an active income, many AIDS patients and their families became financially vulnerable. Thus, a secondary market in life insurance developed to relieve some of the monetary stress of the AIDS victims.[4]

Originally seen by the capital markets as a new financial opportunity, entrepreneurial investors emerged to offer to buy AIDS patients' life insurance policies for a price less than the face value, but more than they could get from lapsing their policy or surrendering it for the cash value. The investors would make the required premium payments (also a difficulty for terminally ill or declining health insureds) and become the beneficiary of the policy (after a "waiting period" had passed). Ultimately, when the insured died, the investor obtained the life insurance policy proceeds. Since AIDS patients were given very little time to live (usually two to three years), the investor did not have many premium payments to make and, after subtracting the initial payment to the insured and subsequent premiums from the final payout of the life insurance policy, the investor could theoretically obtain a large profit (Quinn 2008).

As the financial success of viatical settlement investments became publicized, the secondary market for such life products grew with companies created that specialized in accommodating investors' desires for viatical settlements. It was not much later, however, that this new market collapsed, succumbing to a change in the longevity risk.

Papers presented at the 1996 International AIDS Conference in Vancouver gave evidence of a new drug capable of substantially reducing the level of HIV in those infected (perhaps even to zero). This had a twofold impact: First, it offered new hope for increased life expectancy to the AIDS patients. Second, however, this sudden jump in longevity sounded a death knell for firms that had survived from the profits obtained from viatical settlement sales. This second effect is illustrated by the collapsed value of the viatical settlement firm, Dignity Partner, and by the significant decrease in prices offered to AIDS patients for their insurance policies. With evidence that policies might take a much longer to mature, prices in the viatical market plummeted (Stone and Zissu 2006).

As the viatical settlements market collapsed, investment companies expanded their secondary market life insurance purchases to the elderly to keep the life insurance backed securities asset market alive. Companies chose elderly people with estimated low life expectancies because a low life expectancy meant a greater possibility of profiting sooner from the purchase of life insurance policies. Today, this life settlement market has growing potential[5] as baby boomers are just now entering old age. Additionally, as the population ages, funding retirements over their remaining years of life becomes an escalating concern, especially with increasing simultaneous concerns about funding via Social Security[6].

[4] According to Quinn (2008, p 762), the term “viatical settlement” was coined by Richard Bandfield, a financial planner whose practice assisted the terminally ill. Quinn comments that the term had “a poetic and spiritual definition.” Viatical is from the Latin viaticum, which refers both to Christian communion given to the dying and to provisions given before a journey.

[5] According to Annin, DeMars, and Morrow (2010 p. 1); "It is estimated that in the past five years alone, more than $40 billion of the face value has been sold in the life settlement market."

[6] For example, Couzin-Frankel (2011) relates that every increasing year of life expectancy in the

Life settlements for seniors have also become popular in part due to the extensive marketing pursued by life settlement companies. The senior market now comprises the majority of the entire viatical and life settlements industry. Typically life settlement candidates are over age 65, with some deterioration of health but not terminally ill. They generally have a policy with a death benefit of $250,000 or more and no longer need or can afford the policy (Weber and Hause 2008). Moreover this market may continue to grow. Due to gradual increases in technology and beneficial medical treatment in the United States, the number of centenarians (individuals over the age of 100) has increased from 15,000 in 1980 to roughly 72,000 in 2000 and the number is predicted by the Social Security Advisory Board to reach to 4.2 million, (or approximately 1% of the projected total population) by 2050 (Scotti and Effenberger, 2007). It is estimated that about 50% of individuals born in the USA in the year 2000 will still be alive at age 101 (Vaupel, 2011).

The life settlement market developed at a rapid pace in its early years. A recent survey estimates that the available life settlement market will grow from $13 billion in 2004 to $161 billion over the next few years (Bernstein 2005) through a combination of an aging population, increasing life expectancy and increasing market penetration. Life settlement asset class formation has attracted attention from a broad range of market participants and regulators, including dominant investment banks and major reinsurance companies as intermediaries, the Securities and Exchange Commission (SEC), the National Association of Insurance Commissioners (NAIC) and National Conference of Insurance Legislators (NCOIL), as well as, state regulators, rating agents and life expectancy underwriters.

However, just as advancements in treating AIDS led the viatical settlement market to succumb to longevity risk, the substantial increases in longevity during the 20th and 21st centuries can pose substantial longevity risk to the current life settlement market. A large longevity jump could occur in the future if an effective treatment of coronary heart disease or cancer is found, as these two causes of death combined constitute more than half of all deaths among people over the age of 40 (Johnson, Bengtson, Coleman and Kirkwood 2005 p. 109). Thus, the modeling of longevity risk is of potentially more importance in the life settlements market than it is in the pension market because the life settlement market is based (and funded) on shorter horizons.

2 DESCRIPTION OF THE LIFE SETTLEMENT MARKET

The life settlement market was estimated at $10 billion at 2005, and continued to grow to $12 billion in 2007. Similar to other financial product markets, the life settlement market experienced a contraction during 2008 and the face amount value was estimated at $11.7 billion in 2008 (Conning Research 2008). It still remains attractive as an asset class since as Cox, Lin, and Wang (2006 p.720) explain, life insurance securitization such as found in life settlements are a breakthrough since "it is the first pure mortality security. It stripped out pure mortality risks and thus increased the transparency of the deal."

Weber and Hause (2008) also argue that life insurance assets (such as those involved in life settlements) have sufficiently distinctive characteristics so as to warrant being considered a separate asset class. Some of the distinctive characteristics they describe for this asset class include: 1) The death benefit is cash (a major asset class) provided at the time needed and *without* needing valuation adjustment based on up or down phases of the equity or bond markets; 2) The cash value has asset class attributes, *e.g.*, in a universal or whole life policy the cash value has the dominant characteristic of a fixed account with a minimum guaranteed return while a variable universal life policy's cash value is itself a portfolio reflecting the asset allocation of the policy owner; 3) The life insurance asset has unique tax related characteristics (tax deferred accumulation of cash value, tax-free and possibly estate tax-free death proceeds), the ability to keep policy proceeds out of the reach of creditors, the possibility of using policy cash values

population costs the USA Social Security Administration an additional $50 billion.

to produce retirement income, and the inherent leverage of relatively low periodic payments into a large capital accumulation. These attributes tend to make a life insurance asset uncorrelated with other common asset classes such as equities, fixed income securities, money market funds, etc.. In addition the death benefit is contingent on death and not a capital market event that might cause a change in value, giving it another distinction as an asset class. Rosenfeld (2009) provides further discussion related to life settlements as an asset class.

Before the life settlement market emerged, policy owners had limited choices if they no longer wanted, needed, or could afford the premium payments. Policy owners could cash out a policy by surrendering the policy to the insurance company to receive the surrender value or they could simply stop making premium payments and allow the policy to lapse. In most cases, the policy would be worth considerably more than the surrender value; hence, surrender is an unattractive option. The surrender value is typically based on the commissioner's standard ordinary (CSO) mortality table, in force at the time the policy was issued and so many years before the decision to surrender. These are smooth mortality tables used for conservative non-forfeiture value calculations and do not anticipate extraordinary health changes in individuals, but only aggregate group mortality change characteristics. Thus, if later on the insured has experienced some chronic disease, their mortality profile may be different than that anticipated by the CSO tables and another mortality table may more accurately reflect their anticipated individual mortality probabilities. The cash value calculation is incorporated as part of the insurance contract and is not negotiable, and lapsing the policy forfeits or slowly depletes the cash value in most cases. Under either choice scenario, the extra value in an unwanted or unneeded policy was relinquished to the life insurance company that issued the policy and not captured by the insured. Moreover, prior to the life settlement market development, investors had little access to this asset class other than through their own insurance policies.

A life settlement, however, provides a secondary financial market for this contract and produces an option, other than the surrender or lapsing, to the policy holder. In this way, the policy holder may gain the extra value inherent in the policy rather than relinquishing it to the insurer. When the owner of a life insurance policy no longer needs or wants the policy, the policy is underperforming, the insured can no longer afford to pay the premiums, the business need for the insurance is no longer exists, or a key employee leaves, this secondary market provides the opportunity to resell the policy to a third party for the secondary market price for the policy (*cf.*, Lewis (1989) for a discussion of changes in the need to hold life insurance).

Several market participants and intermediaries play a role in the production of the life settlement; these players include the policy owners, financial advisors or insurance agents, settlement brokers, life underwriters, *i.e.*, who evaluate the life expectancy of the underlying insured life at the time of sale, providers, *i.e.*, parties acquiring the policy and paying the insured for the right to claim the life insurance benefits, and investors, *i.e.*, who either bundle collections of life settlements and securitize them for resale, or keep them for investment purposes as a new asset class in their own portfolio. The majority of investors in today's life settlement market are large institutional investors seeking to acquire large pools of policies which can then be securitized, similar to securitizing mortgages. Retail investors also participate in the life settlement market, generally by purchasing fractional interests in settled policies. To the investor, the life settlement portfolio provides an essentially zero-beta asset which can help diversify a larger portfolio of sensitive financial market assets[7].

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[7] It can also be used as a zero beta asset for valuation of portfolios in a Black type Capital Asset Portfolio Model (Black 1972) instead of the Market portfolio which has well known identifiably problems since Roll's (1977) criticism of the CAPM.

The process or procedures involved in the life settlement transaction are as follows:

1. Insured individuals or policyholders initiate the process to contact a producer, *i.e.,* usually financial advisors or insurance agents. Sometimes the producer contacts the insured because they know the insured needs to sell their life insurance policy.

2. The producer contacts one or more life settlement brokers with a license to do business in life settlements in the policy holders' state of residence since insurance is a state regulated industry.

3. The settlement broker(s) collects the medical information concerning the current health status of the policy holder and "settles" the policy by contacting life expectancy underwriters.

4. The contacted life expectancy underwriters are responsible for preparing a life expectancy assessment and evaluating the mortality risk of the insured based on the current health information provided by the settlement broker.

5. Providers review the data on policy terms, life expectancy, premium amounts, and then bid on the policy. The successful bidder takes over premium payments in return for collecting the ultimate life insurance benefit upon the death of the insured. This bid is based on supplied information and settlement applications prepared by settlement brokers.

6. The existing insured elects to either hold, *i.e.*, not sell in the secondary market, or to sell their policy. If sold the policy can be held in a portfolio or resold to form a life settlement securitization issue which expands the asset class to the broader class of investors with interests in life settlements.

3 PRICING OF LIFE SETTLEMENTS

Two main mathematical methods have arisen for pricing life settlements, a deterministic pricing method and a probabilistic or stochastic pricing method (*c.f.,* Insurance Studies Institute (2008); Zollars, Grossfield and Day (2003); Forman (2010)).[8] The deterministic model is the first and simplest model, and was used almost exclusively in the early days of viatical settlements when life expectancies were short (Zollars, Grossfield and Day 2003). It is still used in some securitization models. We discuss each method in turn

3.1 DETERMINISTIC LIFE SETTLEMENT PRICING

In viatical settlements (and in the early history of life settlement pricing), the life expectancy[9] of the insured was considered the most critical (often the only) variable used in determining the secondary market price of the policy as this represents the expected life length of the insured when the life insurance policy was sold to the third party as a life settlement (the time to payment for the investor). The pricing model was deterministic, like a bond with a payoff at the death of the insured (bond principal equal to the insurance face value) but with negative coupons (premiums) occurring annually until death, a date which was assumed to be the life expectancy with probability one. If T represents the random future life of the insured, then the life expectancy is $\mu = E(T)$. This life expectancy is computed using an appropriate life table, or in the case of life settlements, is usually given by a medical expert based on their examination of the current medical record of the insured. If the discount factor is v =1/(1+r) where r denotes the investors' required rate of return, then the present value of the payoff of a life insurance policy with a benefit of $B is calculated as Bv^{μ}. The premiums paid until the year of death constitute an annuity due (payments at the beginning of the year) and they are subtracted off (in present value) to get the value of the offer. Expenses are further subtracted to arrive at an offer price for the policy. We show below that this deterministic method yields a systematically biased assessment of the value of the payoff and leads to a systematically inaccurate evaluation of the value of the life settlement product.

Theorem 1. The deterministic life settlement pricing model systematically underprices the value of the life settlement benefit in a portfolio of settlements.

Proof: In a portfolio of similar policies where *T* is the (common) time to death, and *B* is the common face value, the expected benefit using the Law of Large

[8] There is also a Monte Carlo simulation approach which is more difficult and less often used. See Zollars et al for details.

[9] Some analysts used the median value (i.e., the value where 50% of a cohort of equivalently rated insureds would die) instead of the mean. Some added a few extra years on for conservativeness.

Numbers is $E(Bv^T)$, with v=1/(1+r). According to Jensen's inequality, if *X* is any random variable, and *f* is any convex function, then *E[f(X)]≥ f(E[X])* with equality if and only if *X* is constant. Taking the random variable as $X=T$, and using the convex function $f(x)=v^x$, yields the inequality

$$\text{Expected Portfolio Benefit Value} \equiv E\left(Bv^T\right) \geq Bv^{E(T)} = \text{Deterministic Value}$$

Taking into account the sequence of premium payments needed to be made prior to death and expenses, the biased result for the deterministic pricing model continues to be exhibited. This is shown below.

Theorem 2. The deterministic pricing model systematically underpays the policyholder for the expected value of their life settlement.

Proof: An annuity of n payments of \$P payable at the beginning of each year starting at time 0 (an annuity due) has a present value at interest rate r equal to $\ddot{a}=\left(1-v^n\right)/rv$ where *v=1/(1+r)* (*c.f.*, Kellison 1991 p 63). The annuity that pays continuously for t periods equals $\bar{a}=\left(1-v^t\right)/\delta$ where is the force of interest, $\delta=\ln(1+r)$, (Kellison 1991 p. 107, Bowers *et. al*., 1986 p. 124), and $\ddot{a}=\left(\delta/rv\right)\bar{a}$. Thus, the present value *X(T)* of the life settlement with face value benefit *B* payable at time *T* and premiums *P* payable at the beginning of each year until the year of death is $X(T)=Bv^T-P\left[\left(\delta/rv\right)\bar{a}\right]=C_1v^T-C_2$, where $C_1=\left[B+\left(P/rv\right)\right]>0$ and $C_2=P/rv$. Since *X(t)* is a convex function of *t*, Jensen's inequality again implies the expected value of the life settlement $E\left[X(T)\right]\geq X\left(E(T)\right)=\text{the deterministic pricing value}$. Hence the deterministic pricing model underprices the policy settlement value to the policyholder. Further subtracting the expenses does not alter this conclusion.

3.2 PROBABILISTIC LIFE SETTLEMENT PRICING

Recognizing the length of life as random variable, we can use standard actuarial mathematics to price the expected life settlement value $E\left(X(T)\right)=E\left(Bv^T-P\ddot{a}\right)$; we only need the probability distribution of T to be able to do so (*cf.*, Bowers et al 1986). The first, and very important, step to this end is to select a mortality table for the life being settled. Different life tables can, as we shall show, produce different values even if they have the same life expectancy.

Information consistent with the medical underwriter supplied assessments must be incorporated into the life table which is to be used. According to Forman (2010) most life settlement companies use some version of the 2008 Valuation Basic Table (VBT), having moved from the 2001 VBT. Other tables are also used (*e.g.* an impaired lives table if the insured is in deteriorated health), and of course the VBT (or any other selected table) most likely will not be consistent with the life underwriters' assessment of life expectancy, so the table must be adjusted in some manner to get to a table which has the required characteristics in order to do the calculation

One standard adjustment approach is to start with a standard table (*e.g.*, the 2008 VBT) and then multiply each mortality rate by a constant factor c selected so as to obtain a new table which produces the underwriter's life expectancy estimate. The derived table terminates when the adjusted mortality rate exceeds 100%. This method is ad-hoc, and while it reproduces the life expectancy, it may not reflect other medical underwriter information. Forman (2010), for example, gives an example of a life expectancy report on a 84 year old woman whose medical records led the underwriter to an estimate of a of 9.2 years of mean life expectancy, a median life expectancy of 9.3 years, and an 85% mortality value of 13 years. The suggested mortality multiplier was 2.03, but this single multiplier may not reproduce all three data points from the starting life table. The report further states "Please note it is recommended that the information provided in this life expectancy evaluation be used in its entirety. If only a subset of the data is used, you will be losing the interrelationships between the analytics." Thus, a methodology must be developed capable of starting with an originating life and then adjusting it to reflect <u>all</u> known information regardless of what internally consistent information is supplied by the underwriter. We discuss how to accomplish this after discussing the choice of originating mortality table.

4 THE CHOICE OF ORIGINAL MORTALITY TABLE PRIOR TO ADJUSTMENT

4.1 THE CHOICE OF A STANDARD TABLE FOR ADJUSTMENT

There is a plethora of life tables available for use, depending on sex, smoking status, health status, retirement status, etc. of the insured life to be settled. As mentioned previously, the 2008 VBT (available from the Society of Actuaries) is commonly used. Other tables include impaired life tables since, as noted by Weber and Hause (2008), the life settlement often involves someone having a deteriorated health status but who is not terminally ill. The selection of an appropriate starting table can improve the accuracy of the end result, even after adjusting it to reflect medical underwriting information.

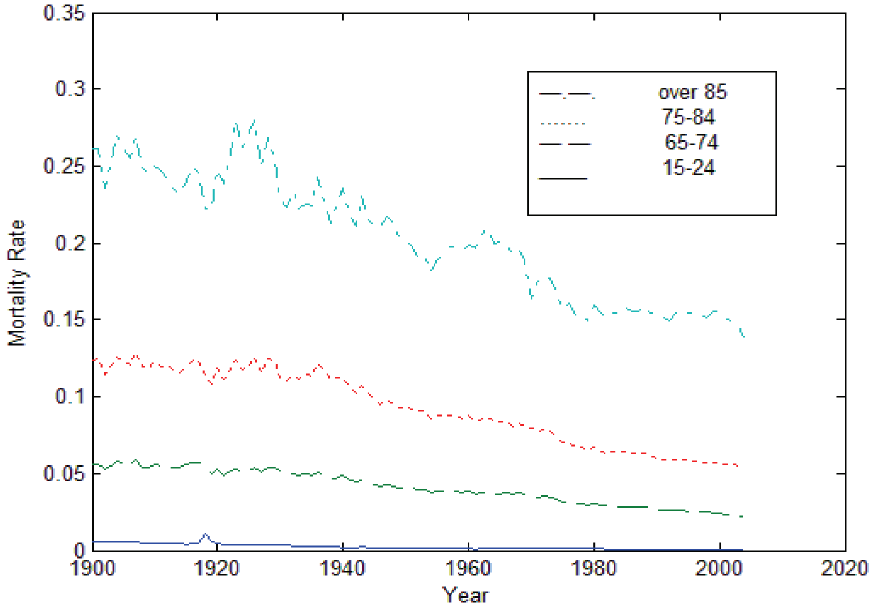
One consideration not explicitly addressed in these mortality tables is the possibility of sudden jumps in mortality or jumps in longevity (such as those that destroyed the viatical settlements market) and their impact on the pricing of life settlements. Jumps in mortality (as opposed to longevity) may also occur (such as an infectious disease that differentially impacts vulnerable elderly populations) and this will increase the internal rate of return on the life settlement for the investor. Currently, jump changes that increase or decrease the expected mortality rate are not incorporated in the mortality models used. Jumps can constitute an important source of return uncertainty in life settlement investments. Below we detail a mortality model which allows stochastic jumps in mortality and longevity, and subsequently we use this model (as well as others) to price life settlements.

4.2 A DOUBLE EXPONENTIAL JUMP DIFFUSION MORTALITY MODEL TO INCORPORATE LONGEVITY RISK

A basic requirement of the mortality model to use for life settlement purposes should be to allow for changes in mortality and longevity rates over time differentially over age groups; this allows the model to anticipate the type of change in medical technology that killed the viatical settlement market or the cures in cardiovascular disease that predominantly effect older insureds. This approach then accommodates additional uncertainty in the financial settlement apart from the evolution of mortality with random departures from an assumed life table.

A collection of models that consider both time and cohort effects are based on the Lee-Carter one-factor model, *i.e.,* see Lee and Carter, 1992. In the Lee-Carter framework, $\mu_{x,t}$ denotes the mortality rate of the group at age is x during the year t. It is decomposed into age-specific parameters a_x and b_x and a mortality trend time-series k_t, using the formula $\ln(\mu_{x,t}) = a_x + b_x k_t + e_{x,t}$ with e denoting a stochastic error term. In this model the a_x vector represents the age mortality pattern of the historical data, k_t represents the improvements over time that have occurred in mortality and b_x represents the improvement rates at age x for general level changes in mortality k_t over time. Using the historical data from HIST290 National Center for Health Statistics, Figure 1 illustrates that the mortality improvement effects do indeed differ by age group but with a common downward trend.

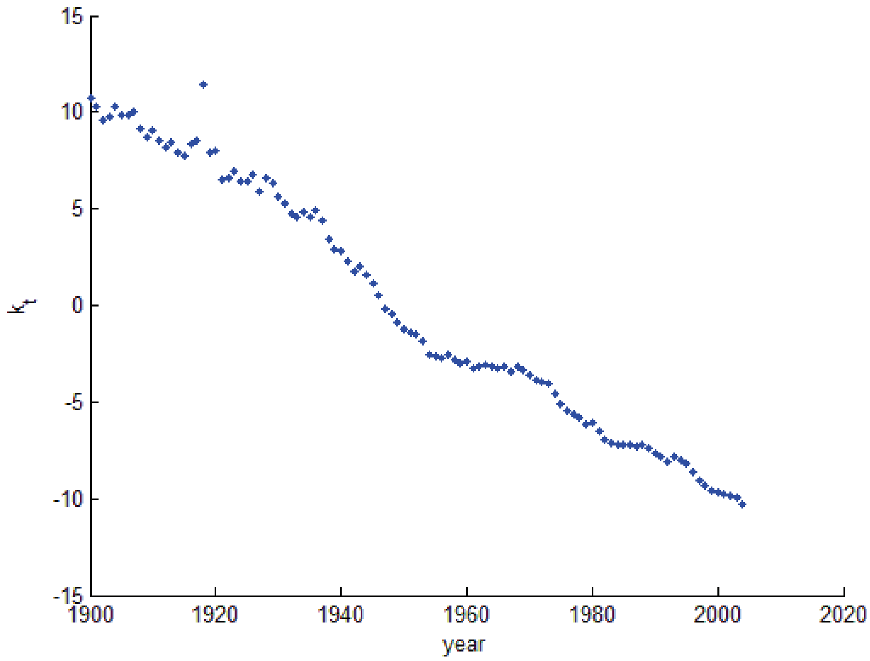
Figure 1, Mortality Rates by Age Group, HIST290 National Center for Health Statistics Data



1900-2004

The common trend over time (k_t) is given in Figure 2.

Figure 2 Trend in mortality (k_t) over years



In this figure, one can observe definite jumps in mortality (such as the 1918 flu pandemic) and in longevity that would have affected returns on life settlements. The extension of the Lee-Carter model and the Chen and Cox (2009) model given in Deng, Brockett and MacMinn (2010) accommodates such jumps. In the Deng, Brockett and MacMinn model, the parameters a_x and b_x are fit as usual using the singular value decomposition method outlined in Lee and Carter (1992). The time varying series (k_t) was modeled as a double exponential jump diffusion (DEJD) detailed below.

The dynamics of the mortality time-series k_t are specified as:

$$dk_t = \alpha\, dt + \sigma\, dW_t + d\left(\sum_{i=1}^{N(t)} (V_i - 1)\right)$$

where W_t is standard Brownian motion, $N(t)$ is a Poisson process with rate λ, and λ describes the expected frequency of the jumps. The larger the λ, the more times jumps occur in the mortality time-series. Here V_t is a sequence of independent identically distributed (iid) nonnegative random variables and $Y = \log(V)$ has a double exponential distribution with the density:

$$f_Y(y) = p\eta_1 e^{-\eta_1 y} 1_{\{y \ge 0\}} + q\eta_2 e^{\eta_2 y} 1_{\{y < 0\}}$$

$\eta_1, \eta_2 > 0$, 膙p $q \ge 0$, 膙$p + q =$.

Age Interval x	*Weighting for Age Interval x*	Age-specific Parameters	
		a_x	b_x
<1	0.013818	-3.4087	0.1455
1-4	0.055317	-6.2254	0.1960
5-14	0.145565	-7.1976	0.1492
15-24	0.138646	-6.2957	0.0994
25-34	0.135573	-5.9923	0.1044
35-44	0.162613	-5.4819	0.0855
45-54	0.134834	-4.7799	0.0608
55-64	0.087247	-4.0137	0.0468
65-74	0.066037	-3.2347	0.0426
75-84	0.044842	-2.4196	0.0409
>85	0.015508	-1.6119	0.0290

Other Parameters	Parameter Value
k_0	-10.302
t	10
α	-0.20
σ	0.31
λ	0.029
γ	-1.25
p	0.035
η_1	0.89
q	0.065
η_2	0.93

The parameters p and q represent respectively, the proportion of positive jumps and negative jumps among all jumps. Thus, $p\lambda$ is the expected frequency of positive jumps and $q\lambda$ the expected frequency of negative jumps. The parameters η_1 and η_2 describe the positive jump size or severity and the negative jump size or severity respectively. Thus, $Y|Y>0$ is exponentially distributed with mean $1/\eta_1$ while $Y|Y\le 0$ is exponentially distributed with mean $1/\eta_2$. In this way, the positive jumps and negative jumps are captured by similar distributions but with different parameters based on the asymmetry of jumps in the mortality time-series k_t and the leptokurtic feature of dk_t. More details of the DEJD and its use in the context of pricing longevity derivatives are given in Deng, Brockett and MacMinn (2010).

The mortality model specification incorporating the double-exponential distribution has the advantage of mathematical tractability allowing a closed-form formula for the expected future mortality rate to be derived (*cf.*, Kou, (2004), Deng, Brockett and MacMinn (2010).[10] The additional parameters of the series k_t are calculated from the observed k_t time series using maximum likelihood (*cf.*, Deng, Brockett and MacMinn (2010), Ramezani and Zeng,, (2007)). The DEJD also fits the data better than the original Lee-Carter model (*cf.*, Deng, Brockett and Macminn (2010)) thus indicating that such jumps have significance. Table 1 gives the parameter values obtained. We apply the DEJD model to generate a mortality table for use in pricing life settlements that also allows for jumps in mortality and longevity.

4 USING INFORMATION THEORY TO OPTIMALLY ADJUST THE STANDARD TABLE

In spite of starting from a seemingly appropriate life table, the medical underwriter's estimate of life expectancy (mean or median) may be incompatible with the dynamics of this table, *e.g.*, the 2001 CSO table may project a 12 year life expectancy for a person whereas the underwriter estimates the expectancy to be 8 years. Information theory provides a rigorous and non-*ad hoc* statistical methodology for adjusting the selected mortality tables so as to incorporate any known individual characteristics into the adjusted table while remaining as close as possible to the original one (*cf.*, Brockett 1991). How to use any known medical information about an individual in underwriting and pricing is a common and important unsettled issue since the security analyst can price life contingent financial instruments (such as life settlements) more accurately with the adjusted table that is consistent with the medical information.

In its simplest form the problem can be summarized as follows: if the medical underwriter has estimated that the expected remaining life is m years, how does one value a life settlement using this information in addition to using a "most appropriate" starting mortality table distribution (which does not have mean m but has other pertinent mortality progression features, such as mortality jumps or being related to disable retired persons). In order to do the pricing using the probabilistic approach, the analyst must construct a mortality table which has $ET = m$, where T denotes the years of life remaining for the life being settled (a random variable). In this section we show explicitly how to obtain an adjusted table that is as indistinguishable as possible from the chosen standard table and that satisfies the underwriter's estimate $ET = m$.

4.1 MINIMUM DISCRIMINATION INFORMATION ESTIMATION

To begin and to develop the intuition for the proposed method, consider the problem of distinguishing or discriminating between two candidate probability densities f and g for some random phenomenon (such as length of life) after observing a value t of the random variable. For example, f and g could correspond to potential densities for the survival time of the individual.

For distinguishing between two densities f and g, the statistic $\ln\left(f(t)/g(t)\right)$ is a sufficient statistic and represents the log odds ratio in favor of the observation having come from f. It can be thought of as the amount of information contained in the particular observation t for discriminating in favor of f over g for modeling the phenomenon (Kullback, 1959). This is the interpretation on which maximum likelihood estimation is based. By the law of large numbers, in a long sequence of observations $\{t_l\}$ from f, the long-run average log odds ratio is:

$$E_f\left(\ln\frac{f(t)}{g(t)}\right) = \sum_l f(t_l)\ln\frac{f(t_l)}{g(t_l)} \qquad (1)$$

which reflects the expected amount of information in an observation for discriminating between f and g. This quantity is called the *divergence* between the densities f and g in the statistics and engineering literature and is denoted by $I\left(f|g\right)$. It is not difficult to show that $I\left(f|g\right) \geq 0$, with $I\left(f|g\right) = 0$ if and only if $f = g$. Thus, the size of $I\left(f|g\right)$ is a measure of the "closeness" of the densities f and g. Such a global measure of divergence between potential probability distributions corresponding to models for the future life random variable will be used for adjusting a standard mortality table to obtain a new "closest" table which reflects the underwriter's information.

[10] One can also use the stochastic mortality model to do Monte Carlo simulation estimates of life settlement values.

To phrase this problem in a general setting, assume we are given a density function *g*, and we wish to find another density *f* that is as close as possible to *g*, and that satisfies *k+1* given expected value or generalized moment constraints involving the expected values of some collection of functions $a_i(t)$:

$$\begin{aligned} 1 &= \theta_0 = \sum f_l \\ \theta_1 &= \sum a_1(t_l) f_l = E_f\left[a_1(T)\right] \\ &\cdots \\ &\cdots \\ \theta_k &= \sum a_k(t_l) f_l = E_f\left[a_k(T)\right] \end{aligned} \quad (2)$$

In the first constraint $a_0(t) \equiv 1$ which simply insures that the *f* is a probability distribution. If we set $a_1(t) = t$ and $\theta_1 = m$ then the second constraint says that the mean for *f* is set to be *m*. As another example, by taking $a_2(t)$ to be unity on a certain interval and zero off the interval, we arrive at a constraint on the probability for that interval, *e.g.*, if the 85 percentile is give as 13 years, then $a_2(t) = 1$ for $t \leq 13$ and 0 otherwise, and $\theta_2 = .85$. The formulation in (2) would also be useful, for example, if one wanted to use a medical study that gives decennial survival probabilities but for which yearly survival probabilities are required. One would then find a survival density that was as close as possible to a standard mortality table and that reflected the decennial survival rates quoted by the medical study. If the median instead of the mean (or both) are given then this can also be expressed in terms of the generalized expectation constraints as in (2). If relative risk values for persons having a particular medical condition (*e.g.*, cardiovascular disease) are given in the medical literature, these can be written in the context of (2) also.

To phrase the problem mathematically, the objective is to find a vector of probabilities $f = (f_1, f_2, \ldots)$ that are as close as possible to the given probability distribution $g = (g_1, g_2, \ldots)$ but which satisfies the moment constraints (2). Written as a mathematical programming problem, we wish to find a collection of probabilities $f = (f_1, f_2, \ldots)$ that solve the problem:

$$\min_f I(f|g) \quad (3)$$

subject to the constraints (2). Here $g = (g_1, g_2, \ldots)$ is the given vector of probabilities corresponding to the standard probability distribution. Brockett, Charnes and Cooper (1980) show that the problem (3) subject to (2) is a convex programming problem and that the dual mathematical programming problem is actually unconstrained and involves only exponential and linear terms (making solving the problem computationally simple). The number of unknowns in the dual is equal to the number of constraints. Moreover, they prove the unique solution has the general form:

$$f_l = g_1 \exp\left[-(\beta_0 + 1) - \beta_1 a_1(t_l) - \cdots - \beta_k a_k(t_l)\right] \quad (4)$$

where the β_i's are constant parameters selected in such a way that the constraints (2) are all satisfied.[11] They further show that the parameters β_i can be obtained easily as the dual variables in an unconstrained convex programming problem:

$$\min_\beta \sum \left\{ g_i \exp\left[-(\beta_0 + 1) - \beta_1 a_1(t_i) - \cdots - \beta_k a_k(t_l)\right] - \left(\beta_0 + \theta_1\beta_1 + \cdots + \theta_k\beta_k\right) \right\} \quad (5)$$

[11] Note from (4) that if we start with a member of the exponential family for g, the resultant f is also of the exponential family of probability distributions. This facilitates estimation and statistical analysis.

T

he solution to (5) can be obtained easily by any number of efficient nonlinear programming codes. In our computations we use Excel Solver.

4.2 ADJUSTING A STANDARD LIFE TABLE TO REFLECT UNDERWRITER INFORMATION

The life expectation used in the life settlement pricing is the expected value of a random variable T that equals the number of years a person now aged x will live.[12] In standard actuarial notation, we have T=0 with probability q_x, T=1 with probability $p_x q_{x+1}$ and, in general, T=k with probability ${}_k p_x q_{x+k}$ where q_x denotes the mortality rate at age x, and ${}_k p_x$ denotes the probability a life age x survives k years, $p_x = 1 - q_x$ and ${}_{k+1}p_x = {}_k p_x\left(1 - q_{x+k}\right)$. Assuming we are given a standard mortality table for an individual age x listing mortality rates at age x, $x+1$, etc., the distribution of the random variable T is given by the $\left(\omega - x + 1\right)$ dimensional probability vector $g = \left(g_0, g_1, \ldots, g_{\omega-x}\right)$, where denotes the end age for the mortality table and the probability of death exactly k years in the future is $g_k = {}_k p_x q_{x+k}$ for $k = 0,1,\ldots,\omega - x - 1$ as calculated from the standard table.

Now consider the problem of finding another mortality table that is as close as possible to the standard table but which additionally satisfies certain given constraints, such as those given in (2). This translates into finding a probability distribution $f = \left(f_0, f_1, \ldots, f_{\omega-x}\right)$ that minimizes (3) for the random variable T and which satisfies the constraint set (2). From the above results the density (4) is the least distinguishable probability density from g among the class of all densities satisfying the constraints.

Let us now illustrate this using the information about the insured most commonly used in life settlements: The medical underwriter has developed an estimate that the curtate expectation of life for the individual whose policy is being settled is m years. Thus, the constraint set for the new table to be used in probabilistic pricing is twofold:

$$1 = \sum f_k, \qquad m = \sum k f_k \tag{6}$$

Appealing to the principle of minimum discrimination information, we select the density f to satisfy:

$$\min I\left(f\,|\,g\right) = \min \sum f_l \ln\left(f_l\,|\,g_l\right)$$

subject to the constraints (6).

We could apply the result (5) directly; however, it is perhaps more instructive to show how to obtain the desired density directly by standard methods in this simple situation. Let $n = \omega - x$. The probability distributions that we are considering can be viewed as $n+1$ vectors $f = \left(f_0, f_1, \ldots, f_n\right)$ that satisfy $f_k \geq 0$, $\sum f_k = 1$ and $\sum k f_k = m$. Letting β_0 and β_1 denote the Lagrange multipliers for the equality constraints (6) allows us to replace the original problem and minimize the function:

$$L(f,\beta) = \sum f_k \ln\left(f_k\,|\,g_k\right) - \beta_0\left(1 - \sum f_k\right) - \beta_1\left(m - \sum k f_k\right)$$

subject to $f_k \geq 0, k = 1,\ldots,n$. The $n+3$ first-order conditions found by differentiating with respect to $f_0, f_1, \ldots, f_n, \beta_0, \text{and}\, \beta_1$ are as follows:

$$\ln\left(f_k\,|\,g_k\right) + 1 + \beta_0 + k\beta_1 = 0,\ k = 0,\ldots,n;$$

$$-1 + \sum f_k = 0;$$

$$-m + \sum k f_k = 0$$

The first $n+1$ equations give $f_k = g_k \exp\left(-1 - \beta_0 - k\beta_1\right)$ for $k = 0,\ldots,n$. The last two equalities allow the determination of the parameters $1 + \beta_0$ and β_0 .

Consider the function $\Phi(\beta) = \sum g_k e^{-k\beta}$.

Since $\sum f_k = 1$, we have $1 = \sum g_k\, e^{-1-\beta_0-k\beta_1} = e^{-1-\beta_0}\,\Phi(\beta_1)$. Therefore, $1 + \beta_0 = \ln\Phi(\beta_1)$. Thus, if we can find β_1 then we can also determine β_0 .

[12] Some underwriter use the median instead of the mean, but as noted previously this is still of the form (2) and can be readily accommodated. We shall, however, use the mean value in our illustrative analysis.

To determine β_1 note that $\Phi'(\beta_1) = -\sum g_k k e^{-k\beta_1}$, so that

$$\Phi'(\beta_1) = -\sum k g_k e^{-k\beta_1} = -e^{(1+\beta_0)} \sum k g_k e^{1-\beta_0-k\beta_1}$$

$$= -e^{(1+\beta_0)} \sum k f_k = -e^{(1+\beta_0)} m = -\Phi(\beta_1) m$$

Hence, in order to find the precise numerical value for β_1, we solve:

$$\Phi'(\beta_1) = -\Phi(\beta_1) m$$

or equivalently:

$$\frac{d}{d\beta} \ln\left(\Phi(\beta)\right) = -m.$$

This can be done by any of a number of software programs (*e.g.*, we used Excel Solver). Obtaining β_0 through the equation $1+\beta_0 = \ln \Phi(\beta_1)$ yields both parameters β_0 and β_1 from which we readily calculate the desired adjusted probability distribution $f_k = g_k e^{-(1+\beta_0+\beta_1 k)}$.

5 ILLUSTRATIONS AND COMPARISONS OF LIFE SETTLEMENT PRICING

In this section we shall present numerical illustrations of the previous information theoretic probabilistic pricing of life settlements. Comparisons are made across different assumed "standard" starting tables. For compatibility of the illustrations across the various numerical computations, we shall use the same person whose life is being settled in all examples. We assume that a State Farm Insurance Company Whole Life policy paid up at age 100 was issued on a female standard risk non-tobacco user in good health at age 40 in 1986 who is age 70 in 2006, the year of settlement. To keep the numbers reasonable and to conserve space, we assume the insurance policy has a face value (death benefit) of $50,000. The policy details are adapted from Appendix C in Baranoff, Brockett and Kahane (2009) and are listed in Table 2.

Table 2. Policy Details

Face Amount	Annual Premium	Guaranteed Cash Value (Using 2001 CSO Mortality Table)
$50,000	$565.50	$8,438.50 at age 70

5.1 ILLUSTRATION OF ADJUSTING A MORTALITY TABLE TO REFLECT INFORMATION

For the purposes of illustration, we assume that the medical underwriter has estimated that the life expectancy of the insured is 8.5 years. We assume a uniform distribution of deaths during the year of death. Considering that the 2001 CSO table upon which the policy cash values were predicated predicts a life expectancy of 16.4 years for a 70 year old female, the individual is in somewhat of a deteriorated but not life threatening condition.

To illustrate the adjustment process we start with a standard life table designed for disabled retired lives.[13] Table 3 presents the mortality rates for this standard table starting with the settlement age of 70. The expectation of life for this individual using the disabled retired lives table is 12.9 years, so the mortality rates must be adjusted up to create a new table consistent with the underwriter's assessment of 8.5 years.[14] The death year probabilities g_k are calculated from the mortality rates q_k as previously described and are given in the third column of Table 3.

Adopting the information theoretic approach to adjust the standard table, we present in the fourth column of Table 3 the adjusted death year probabilities f_k calculated as the solution f_k which minimizes $I(f|g)$ subject to

$$1 = \sum f_k, \quad \text{and} \quad 8.5 = \sum (k+.5) f_k$$

(the .5 term in the second summation arises because of the uniform distribution of

[13] Available from the Society of Actuaries at www.soa.org/files/pdf/rp00_mortalitytables.pdf

[14] If instead of having estimated the mean as 8.5 the underwriter had specified that he thought the mean was between 6.5 and 8.5, the resulting adjusted mortality table turns out to be the same as that obtained using the single equality constraint, even though the inequality constrained table has one more parameter to estimate.

deaths assumption). As derived previously, the solution is of the form

$$f_k = g_k \exp\left(-1-\beta_0 - k\beta_1\right),$$

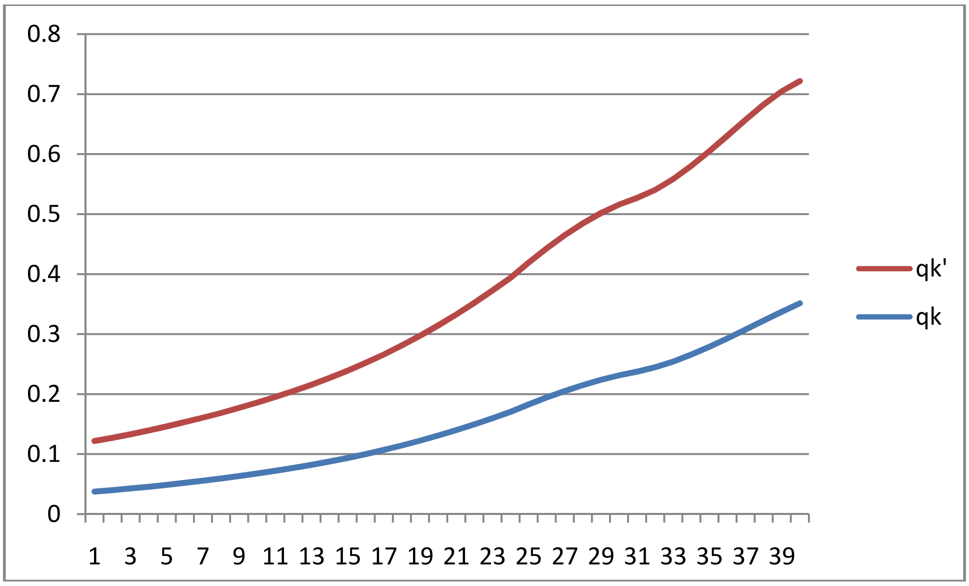
and the computation described earlier produces $_0$ = *-1.80579* and $_1$ = *0.080089*. Thus, $f_k = 2.2384\ g_k(1.083384)^k$. The resulting mortality rates for the adjusted table are denoted by $q_x{}'$. While the computations are carried out to the end of the life table (age 115), Table 3 only presents the results for the first 20 years to preserve space. The final column shows the adjusted mortality rates.

Table 3. Standard (Disabled Retired) Life Table and Adjusted Life Table That Achieves a Life Expectancy Equal to 8.5 Years

	A ge	Disable Retired Mortality Rate	Disable Retired Probability of Death in Year	Adjusted Table Probability of Death in Year	Adjusted Table Mortality Rate
		q_k	g_k	f_k	$q_k{}'$
	7	0.0376	0.037635	0.0842449	0.0842449
	7	0.0401	0.0386293	0.0798154	0.0871580
	7	0.0428	0.0395829	0.0754911	0.0903069
	7	0.0457	0.0404667	0.0712367	0.0936772
	7	0.0488	0.0412520	0.0670298	0.0972557
	7	0.0522	0.0419111	0.0628593	0.1010304
	7	0.0557	0.0424196	0.0587253	0.1049937
	7	0.0595	0.0427594	0.0546396	0.1091489
	7	0.0635	0.0429147	0.0506174	0.1135027
	7	0.0677	0.0428742	0.0466775	0.1180693
	8	0.0723	0.0426318	0.0428413	0.1228734
	8	0.0771	0.0421868	0.0391312	0.1279548
	8	0.0822	0.0415387	0.0355645	0.1333556
	8	0.0878	0.0406863	0.0321536	0.1391178
	8	0.0937	0.0396289	0.0289076	0.1452852
	8	0.1002	0.0383659	0.0258322	0.1518975
	8	0.1070	0.0368973	0.0229313	0.1589898
	8	0.1145	0.0352260	0.0202076	0.1665921
	8	0.1224	0.0333582	0.0176634	0.1747249
	8	0.1309	0.0313068	0.0153012	0.1834039
	9	0.1400	0.0290920	0.0131244	0.1926435

To illustrate the effect of the information theoretic adjustment, Figure 3 shows the unadjusted and adjusted mortality rates. The optimal adjustment is not simply a multiple as is often used in life settlement pricing.

Figure 3. Mortality Rates For a 70 Year-old Using the Disabled Retired Life Table Rates q_k and Then Adjusting Them to Obtain New Mortality Rates q_k' That Make The Life Expectancy Equal 8.5 Years



5.2 LIFE SETTLEMENT PRICING USING SEVERAL STARTING TABLES

The first and simplest pricing model is the deterministic model which assumes that the benefit payment occurs at the predicted life expectancy date with probability 1, *i.e.*, the mortality rates in the mortality table are zero for all dates other than the precise expected death date, at which point the mortality rate is one. As mentioned previously, pricing using this deterministic method is like pricing a bond with principal (face value) paid at the date of expected death and the coupons being negative and in value equal to the specified premium. For deterministic life settlement pricing of the policy specified in the previous section having a life expectancy of 8.5 years, the present value of the future benefit is $\$50{,}000\times\left(1/(1+r)\right)^{8.5}$ and the premium payments constitute an annuity due for 8 years of amount \$565.50 at the beginning of each year. Subtracting the annuity of premium payments from the discounted benefit value yields the deterministic value. Further subtracting expenses yield the price to be paid.

We next turn to the probabilistic pricing model and investigate the sensitivity of the life settlement value obtained from the adjusted table to the choice of the starting standard mortality table used. Again adjustment is made so that the adjusted table in each case has an expected life of 8.5 years for the 70 year old considered previously. For each starting table we solved the information theoretic optimization problem to find the parameters $_0$ and $_1$ needed to adjust the table to obtain the mean of 8.5, and then calculated the expected present values for this adjusted table using the formula $X(T)=Bv^T-P\ddot{a}$ and using the adjusted table probability distribution for T. Our choice of starting tables includes the Disabled Retiree table discussed previously, a Healthy Annuitant life table,[15] the 2001 CSO Table used to establish the cash values, the 2001 VBT table,[16] the 2008 VBT table[17] and finally the DEJD mortality model presented earlier which allowed for historically observed potential jumps in mortality and longevity over the years. Table 4 presents the results of calculating the life settlement value using the formula $X(T)=Bv^T-P\ddot{a}$ starting from different tables and using differing internal rates of return r. Figure 4 displays these results graphically.

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[15] Available from www.soa.org/files/pdf/rp00_mortalitytables.pdf

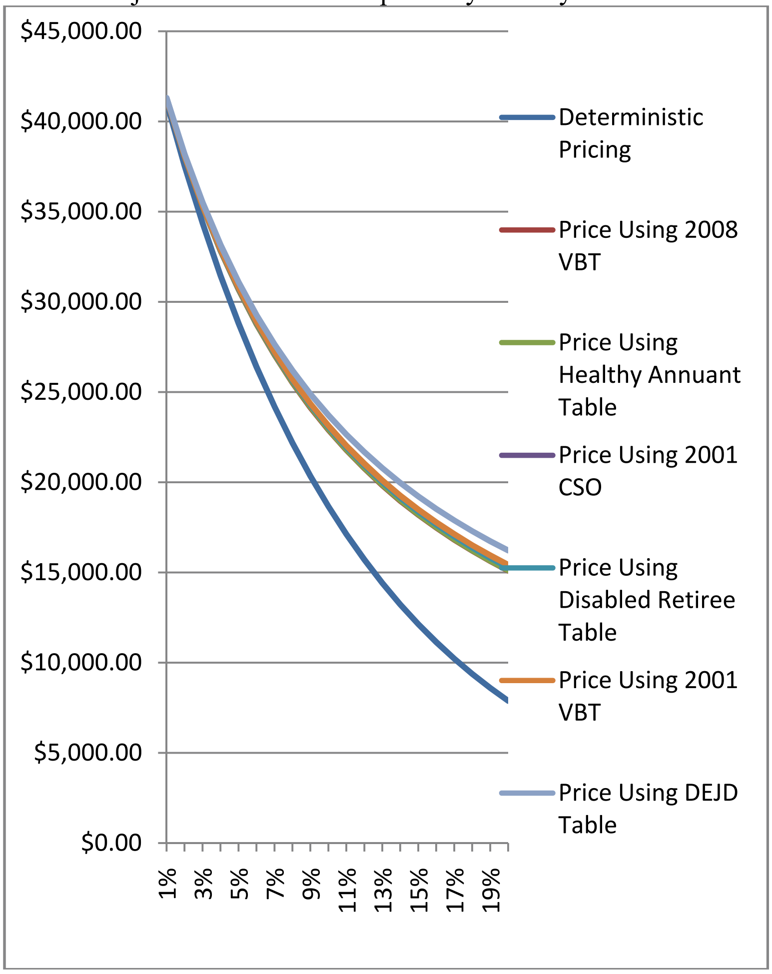
[16] The 2001 CSO table and the 2001 VBT table are available from the Academy of Actuaries in Appendix A located at http://www.actuary.org/life/CSO_0702.asp

[17] The 2008 table and report are available from www.actuary.org/pdf/life/tables_march08.pdf

Table 4. Present Value of the Life Settlement Starting from Different Mortality Tables All Adjusted to Have Life Expectancy of 8.5 years

Assumed Rate of Return	Deterministic Pricing	Price Using 2008 VBT	Price Using Healthy Annuitant Table	Price Using 2001 CSO	Price Using Disabled Retiree Table	Price Using 2001 VBT	Price Using DEJD Table
1.00%	$41,282	$41,479	$41,482	$41,489	$41,491	$41,493	$41,531
2.00%	$37,966	$38,489	$38,496	$38,515	$38,519	$38,525	$38,622
3.00%	$34,935	$35,871	$35,883	$35,916	$35,920	$35,933	$36,099
4.00%	$32,162	$33,566	$33,583	$33,632	$33,637	$33,656	$33,895
5.00%	$29,622	$31,529	$31,550	$31,614	$31,619	$31,646	$31,957
6.00%	$27,293	$29,718	$29,743	$29,823	$29,826	$29,863	$30,242
7.00%	$25,158	$28,103	$28,130	$28,226	$28,227	$28,273	$28,718
8.00%	$23,198	$26,656	$26,685	$26,796	$26,794	$26,849	$27,354
9.00%	$21,398	$25,354	$25,384	$25,509	$25,504	$25,569	$26,128
10.00%	$19,743	$24,178	$24,209	$24,348	$24,339	$24,413	$25,021
11.00%	$18,221	$23,112	$23,144	$23,295	$23,282	$23,365	$24,018
12.00%	$16,819	$22,144	$22,175	$22,337	$22,320	$22,412	$23,104
13.00%	$15,529	$21,260	$21,291	$21,463	$21,441	$21,542	$22,270
14.00%	$14,339	$20,451	$20,481	$20,663	$20,637	$20,746	$21,504
15.00%	$13,242	$19,709	$19,737	$19,928	$19,897	$20,015	$20,800
16.00%	$12,229	$19,025	$19,052	$19,251	$19,216	$19,341	$20,150
17.00%	$11,294	$18,395	$18,420	$18,626	$18,586	$18,718	$19,548
18.00%	$10,431	$17,811	$17,834	$18,048	$18,003	$18,142	$18,990
19.00%	$9,632	$17,270	$17,291	$17,511	$17,461	$17,607	$18,471
20.00%	$8,893	$16,767	$16,786	$17,011	$16,958	$17,109	$17,986

Figure 4. Comparison of Life Settlement Values Starting from Different Mortality Tables Adjusted to have Life Expectancy of 8.5 years



It is worth noting that after information theoretic adjustment, Table 4 shows definite differences between the values obtained using different starting tables, however for this low value ($50,000 face) policy for the most part these differences appear relatively small. For larger policies, the difference in price can be significant. Additionally, according to the Society of Actuaries Life Settlements Survey Task Force (SOA 2010), the median size of the face value of settled policies in some product lines, such as Universal Life Policies with Secondary Guarantees, is over $1,000,000, with some companies reporting an average size for their settled policies of over $3,000,000. The magnitude of the difference between adjusted values by starting table chosen also becomes more pronounced at higher internal rates of return, and Murphy (2006) estimated that the rate of return could be in the range 15-18%. Thus, choice of starting table is important.

It is also worth noting that as expected (and proven) the deterministic method always under prices relative to all of the mortality tables used, even though it also has the same life expectancy. Also it is worth noting that the adjusted DEJD model gives the largest value. Perhaps this later result is due to the DEJD model allowing for both jumps in mortality and in longevity, with unexpected mortality jumps historically occurring more frequently. Since mortality jumps increase returns to the life settlement investor, investors should be willing to pay more to purchase under this model than under models which do not anticipate any potential for such jumps.

6 EXTENSIONS AND CONCLUSION

This paper showed that the commonly used deterministic method for pricing life settlements is systematically biased, and that the probabilistic method is superior. The paper then presented an easily implemented method for adjusting a mortality table to exactly reflect information useful for pricing life settlement products using a probabilistic or stochastic mortality methodology in a non-ad-hoc and statistically rigorous manner.

The results of this paper can be extended in several directions. First, the amount of information that can be incorporated into the information theoretic adjustment process can be substantially more than just the life expectancy we used in our illustration. As noted earlier, Forman (2010) gives a life settlement underwriter's report estimating not only the mean but also the median and 85% of the projected life distribution. All of these can be incorporated into the adjustment process using the information theoretic optimization methodology of this paper. Additionally, due to our mathematical programming formulation, inequalities can be input as constraints as well. For example, one may specify that the mortality rates in the adjusted table are monotone; alternatively concavity in the rates at older ages may be specified. Finally smoothness can be imposed if desired. See Brockett (1991)and Brockett and Cox (1984) for guidance on how this might be done. Finally, the information theoretic approach can be applied to mortality rates themselves rather than mortality probabilities if so desired (the mathematics does not depend on the objective function and constraint sets involving probability measures, only nonnegative quantities). Constraints on the adjusted mortality rates may be more easily incorporated in this formulation.

An important additional problem faced by life settlement medical underwriters when attempting to furnish mortality estimates for the life settlement industry is how to incorporate other medical study results on potential infirmities into their life value estimate. Using the information theoretic approach outlined here the relative risk of a particular disease found in the medical literature can be incorporated as a constraint as well. This constraint would specify that the expected number of deaths in the adjusted table is a given multiple (the relative risk) of the number of deaths expected by the table used for comparison purposes in the medical study (which need not be the same table as the one the analyst is using as a starting point for creating an adjusted table for their life settlement

pricing use).

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